

Periodic thick-brane configurations and their stability

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I investigate models with scalar fields in 5 dimensions that exhibit thick-brane configurations with a non-trivial metric. I show that an appropriate coupling to the scalar curvature allows for periodic configurations, which, however, are unstable under small harmonic perturbations. A model for stabilizing these configurations is proposed and discussed.

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I. INTRODUCTION

One of the most interesting proposed extensions of the standard model is based on a 5-dimensional space-time [1] containing one or more branes, 4-dimensional subspaces where some of the fields are localized. Such models usually assume that space time has a topology $\mathbb{M} \times \mathbb{B}$, where \mathbb{M} denotes the usual Minkowski space and \mathbb{B} is a manifold that may or may not be compact, and a non-separable metric (I consider here only the so-called “small extra dimension” models); the original models assumed \mathbb{B} to be one-dimensional but this has since been generalized [3]. When this space-time configuration and brane content correspond to a stable solution of the field equations, it can serve as a vacuum where quantum fluctuations propagate; the assumption being that this background configuration corresponds to a minimum of the effective action for the complete system.

This paradigm offers an innovative solution to the hierarchy problem and a wealth of new effects that may be observed at the LHC. Motivated by this, several authors have constructed realistic or semi-realistic models based on this idea, and have studied a variety experimental signatures [2].

The stability of the background configurations under small harmonic perturbations has also been studied [1, 7, 25]. In the seminal publications [1] it was shown that for the models there considered most perturbations are not destabilizing; the one exception being the dilaton, a self-similar scaling perturbation which is neutral (has no quadratic potential term or, equivalently, is associated with zero frequency). This problem, however can be eliminated by the addition of appropriate bulk scalar fields [8], which has been studied in a variety of cases: for single scalar fields coupled minimally to gravity [8, 9], non-minimally coupled scalars [10], and for Brans-Dicke theories [11].

These higher-dimensional theories, being non-renormalizable, have an intrinsic ultraviolet cutoff scale Λ beyond which they are not reliable (at least within perturbation theory). In models where branes are infinitely thin (in the fifth coordinate), it is tacitly assumed that their structure will become manifest at scales $\gtrsim \Lambda$. But this need not be the case, and a certain amount of attention has been paid to the possibility that the dynamics at scales below Λ is responsible for the brane configuration. A variety of such models have been studied in the literature for non-compact \mathbb{B} with one [12–14], or several [15, 16] minimally-coupled scalars. In addition, the stability of such models has been investigated for the case of non-compact \mathbb{B} and one [13, 14] or several [16] minimally coupled scalars, and for \mathbb{B} compact and a flat metric [18].

Models with \mathbb{B} compact that exhibit periodic, stable, brane-like configurations generated by physics below Λ (“thick branes”), and which have a non-trivial metric are more difficult to construct. The reasons are, first, that for the simplest models the background configurations cannot satisfy the periodicity requirement (*e.g.* they fail to satisfy one or more of the sum rules listed in [17], as required for consistency). And second, general considerations [18, 19] apparently preclude the stabilization of thick-brane solutions using only scalar fields.

The goal of the present paper is to provide mechanisms that overcome these two difficulties. Specifically, to exhibit a class of 5-dimensional models containing gravity, scalars, and antisymmetric tensor fields, which admit periodic kink-like solutions that are perturbatively stable at scales below Λ . Though the theories thus obtained have no phenomenological applications (*e.g.* they exhibit but a small amount of warping, so that no significant mass hierarchy can be generated), they are of interest because they can address these two problems. Though there are some indications that more realistic configurations can also be obtained (sect. II A), I will not attempt to construct a phenomenologically viable model, nor will I not attempt to address the much more ambitious and difficult problems

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of confining fields to the branes and of global stability. For a different class of theories, based on the “large extra dimensions” paradigm [5], there are realistic models that include both dynamically generated branes and a confining mechanism for the Standard Model fields; see for example [6].

The calculations presented are essentially classical, amounting to obtaining solutions to the equations of motion that are stable under harmonic perturbations. The usefulness of these results lie in the well-established connection between such solutions and related quantum objects [20]; in particular, the classical background solutions are to be interpreted as the lowest order semi-classical approximation to a quantum vacuum, and the frequencies of the harmonic perturbations as the energies of the low-lying excitations.

The plan of this paper is the following. The next section introduces a class of models involving only scalars, that support periodic configurations, which in favorable cases are similar to the ones in [1]. These background configurations are unstable (Sect. III), but this problem can be solved through the introduction of antisymmetric tensor fields (Sect. IV), adequately coupled. The last section contains some parting comments and observations, while some mathematical considerations are delegated to the appendices.

II. THE MODEL (BASIC VERSION)

I first consider the problem of constructing kink-like configurations that involve gravity and are periodic in the fifth coordinate of a 5-dimensional space-time¹. Though it is well known [22] that scalar models with Lagrangians of the type $\mathcal{L} \sim (\partial\phi)^2 - V(\phi)$ do allow stable kink and multi-kink configurations, none of these satisfy the periodicity constraint (the simplest way of seeing this is by noting that such configurations violate one or more of the sum rules of Ref [17], as shown in that same publication); fortunately a simple and natural modification overcomes this obstacle. I consider models of the type [10],

$$\mathcal{L} = 2M_{\text{Pl}}^3 R - \frac{1}{2}g^{ij}\tilde{\phi}_{r,i}\tilde{\phi}_{r,j} - \frac{1}{2}\xi_{rs}(\tilde{\phi}_r + \hat{n}_r\varphi)(\tilde{\phi}_s + \hat{n}_s\varphi)R + V(\tilde{\phi}) , \quad (1)$$

where M_{Pl} denotes the 5-dimensional Planck mass, and R the scalar curvature, a comma denotes an ordinary derivative, $\tilde{\phi}$ is an N component real scalar field with components $\tilde{\phi}_r$, and ξ is a real, symmetric $N \times N$ matrix. The vector \hat{n} denotes a constant direction in field space, while φ is a scale of the same order as $M_{\text{Pl}}^{3/2}$. The sign of the potential term is chosen for later convenience.

Models of the type (1) have been studied in the literature, in particular Ref. [18] provides flat-space, single-field examples where the background solutions are both periodic and stable; unfortunately these models do not extend easily to the case of a non-trivial metric.

I assume now that M_{Pl} is the largest scale in the theory and consider solutions that have a small but non-trivial deviation from a flat metric (recalling that $\varphi \sim M_{\text{Pl}}^{3/2}$):

$$\begin{aligned} \tilde{\phi} &= \phi + O(1/\varphi); \\ g_{ij} &= \eta_{ij} + \frac{1}{\varphi}h_{ij} + \frac{1}{\varphi^2}k_{ij} + O(1/\varphi^3); \end{aligned} \quad (2)$$

where $\eta_{ij} = \text{diag}(-1, 1, 1, 1, 1)$ denotes the flat-space metric. Substituting in (1) gives, after some algebra,

$$\begin{aligned} \sqrt{-g}\mathcal{L} &= -\frac{1}{2}\sum m_{rs}\phi_{r,i}\phi_{r,i} + V(\phi) \\ &\quad - \frac{8}{3\xi}\left[\bar{\xi}\hat{\mathbf{a}} \cdot \phi_{,i} - \frac{3}{16}(\mathbf{a} \cdot \hat{\mathbf{n}})q_i\right]\left[\bar{\xi}\hat{\mathbf{a}} \cdot \phi^{,i} - \frac{3}{16}(\mathbf{a} \cdot \hat{\mathbf{n}})q^i\right] \\ &\quad + \frac{|\mathbf{a}|^2}{8\xi}\left(h^{ij,k}h_{ij,k} - h^{ik},_kh_{ij},^j - \frac{1}{4}q_iq^i\right) + O(1/\varphi), \end{aligned} \quad (3)$$

where space-time indices are raised and lowered using the flat metric η , and where

$$q_i = h_{i,k}^k - h_{k,i}^k, \quad \mathbf{a}_r = \xi_{rs}\hat{n}_s,$$

¹ In the following i, j , etc denote 5-dimensional space-time indices, Greek indices will refer to the 4-dimensional non-compact coordinates; the metric signature is $(-1, +1, +1, +1, +1)$. I use the conventions of Landau and Lifshitz [21] for the definition of the Riemann and associated tensors. I denote the compact coordinate by y , and the non-compact ones by x^μ , $\mu = 0, 1, 2, 3$. The indices r, s , etc. label the scalar-field components.

$$m_{rs} = \delta_{rs} - \frac{16}{3} \bar{\xi} \hat{\mathbf{a}}_r \hat{\mathbf{a}}_s, \quad \bar{\xi} = \frac{|\mathbf{a}|^2}{\mathbf{a} \cdot \hat{\mathbf{n}} - 4M_{\text{Pl}}^3/\varphi^2}. \quad (4)$$

The corresponding equations of motion are:

$$\begin{aligned} m_{rs} \square \phi_s + \left(\frac{\partial V}{\partial \phi_r} \right) &= 0; \quad \square = \eta^{ij} \partial_i \partial_j, \\ \square h_{ij} - h_{i,jk}^k - h_{j,ik}^k + h_{k,ij}^k &= -4 \frac{\bar{\xi}}{|\mathbf{a}|} \left[(\hat{\mathbf{a}} \cdot \phi)_{,ij} + \frac{1}{3} \eta_{ij} \square (\hat{\mathbf{a}} \cdot \phi) \right]; \end{aligned} \quad (5)$$

which can also be obtained by expanding the Einstein and field equations and using (2).

A. Background solutions

With the goal of preserving 4-dimensional Lorentz invariance I look for background configurations of the form

$$\begin{aligned} \phi &= \Phi(y), \\ h_{ij} &= H_{ij}(y), \quad H_{\mu\nu} = -2\sigma(y)\eta_{\mu\nu}, \quad H_{4\mu} = H_{44} = 0; \\ k_{ij} &= K_{ij}(y), \quad K_{\mu\nu} = -2\tau(y)\eta_{\mu\nu}, \quad K_{4\mu} = K_{44} = 0; \end{aligned} \quad (6)$$

which solve (5) to lowest order in φ provided

$$\begin{aligned} \sigma &= \frac{2}{3} \frac{\hat{\mathbf{a}} \cdot \Phi}{\hat{\mathbf{a}} \cdot \hat{\mathbf{n}}} + \text{const.}, \quad \Phi' m \Phi' = \left[-\frac{27}{2} (\mathbf{a} \cdot \hat{\mathbf{n}}) \sigma^2 + \Phi \xi \Phi - 3(\mathbf{a} \cdot \hat{\mathbf{n}}) \tau \right]'', \\ \frac{1}{2} \Phi' m \Phi' + V(\Phi) &= 0, \quad m \Phi'' = \mathbf{F}; \quad F_r = - \left(\frac{\partial V}{\partial \phi_r} \right)_{\phi=\Phi}; \end{aligned} \quad (7)$$

where a prime denotes a y derivative.

The last two equations in (7) describe the zero-energy, classical, non-relativistic motion in a potential V ; the mass (cf. eq. 4) equals one except when Φ is parallel to \mathbf{a} , in which case the mass is $1 - (16/3)\bar{\xi}$ (for a different interpretation when $\bar{\xi} > 3/16$, see A). Higher-order corrections (in $1/\varphi$) can be determined similarly, but the expressions are cumbersome; an example is presented in B.

The warp factor in this type of models is $\sim 1 - 2\sigma/\varphi$, and cannot generate a significant mass hierarchy. Note however that the expression in B for τ shows that this function is also periodic, with the same period as σ ; which suggests that periodic solutions exist also for moderate values of φ . Verification of this conjecture, as well as a determination of the range in φ for which it holds, lies beyond the scope of this paper.

In the following I will choose coordinates in field space such that

$$\hat{\mathbf{a}}_r = \delta_{r,1} \Rightarrow m = \text{diag} \left(1 - \frac{16}{3} \bar{\xi}, 1, 1, 1, \dots \right), \quad (8)$$

and will denote by m_r the eigenvalues of m :

$$m_1 = 1 - \frac{16}{3} \bar{\xi}, \quad m_r = 1, \quad 2 \leq r \leq N \quad (9)$$

A judicious choice of the potential V and initial conditions will lead to solutions that are periodic in y ,

$$\Phi(y + L) = \Phi(y), \quad (10)$$

with L determined by V and the initial conditions; σ will also be periodic with the same period. For example, adopting the basis (8) and taking

$$V(\phi) = \sum_r m_r u_r(\phi_r), \quad (11)$$

the equations (7) yield

$$\sigma = 2\Phi_1/(3\hat{n}_1); \quad \Phi_r'' = f_r(\Phi_r); \quad (12)$$

where $f_r = -du_r(\phi_r)/d\phi_r$ (no sum over r). An appropriate choice of u_r and initial conditions will then generate solutions obeying (10).

Adopting such solutions as background configurations allows the identification $y \equiv y \bmod L$, which amounts to a compactification of the fifth dimension. Such configurations can serve as vacua for the theory, provided they are stable.

B. Sum rules

It is of interest to see how the present models avoid the obstacles listed in Ref. [17]. The most severe constraint is obtained by assuming the existence of periodic solutions and integrating the second equation in (7) over a period:

$$\oint dy \Phi' m \Phi' = 0. \quad (13)$$

When $\bar{\xi} = 0$ this can be satisfied only when $\Phi' = 0$ (in the presence of localized branes there are additional contributions and the sum-rule can be satisfied by non-trivial Φ configurations [17]). In contrast, when $\bar{\xi} \neq 0$, non-trivial solutions are allowed provided m is not positive definite; from (8) this corresponds to

$$\bar{\xi} > \frac{3}{16}, \quad (14)$$

which I assume henceforth. In this case the mass matrix (8) takes the form

$$m = |m|S; \quad S = \text{diag}(-1, +1, +1, \dots). \quad (15)$$

C. Comparison with the 2-brane Randall-Sundrum model

In the 2-brane Randall-Sundrum model [1] the background of the form (6) is also adopted, but instead of introducing scalar fields it is assumed that the space-time contains two branes with cosmological constants $\pm\lambda$, and a bulk cosmological constant $\Lambda < 0$. The resulting field equations are

$$24M_{\text{Pl}}^3 \sigma'^2 = -\Lambda \varphi^2, \quad 12M_{\text{Pl}}^3 \sigma'' = \lambda \varphi [\delta(y) - \delta(y - L/2)], \quad (16)$$

where $0 \leq y \leq L$ is the range of the compact coordinate.

In order to compare this to the previous results I again adopt (8) and assume the potential takes the form (11). Then, from (7),

$$24M_{\text{Pl}}^3 \sigma'^2 = \frac{32M_{\text{Pl}}^3}{3\hat{n}_1^2} \Phi_1'^2, \quad 12M_{\text{Pl}}^3 \sigma'' = \frac{8M_{\text{Pl}}^3}{\hat{n}_1} \Phi_1''. \quad (17)$$

Using (12), it is clear that for a very flat potential u_1 (see Fig. 1), Φ_1' will be almost constant except at the turning points where it will rapidly drop to zero, while Φ_1'' will be almost zero except at the turning points, where it will be large. This type of potential then yields configurations qualitatively similar to those derived from (16) for the case $\Lambda < 0$.

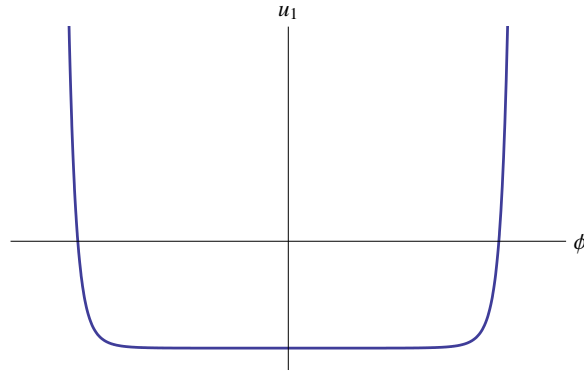


FIG. 1: Scalar potential leading to configurations similar to those described in [1]

III. STABILITY

The usefulness of the above solutions as background configurations depends on their stability; the minimal requirement being that all periodic solutions to the linearized perturbation equations are bounded in time, *up to coordinate transformations*. As usual, such linear perturbations can be assumed to depend harmonically on the non-compact coordinates.

The relevance of coordinate transformations can be illustrated by considering a background solution of period L ,

$$\Phi(y) = \sum \mathbf{f}_n e^{2\pi i n y / L}; \quad \sigma(y) = \sum \mathbf{s}_n e^{2\pi i n y / L}; \quad (18)$$

and a perturbation that consists in the replacement $L \rightarrow (1 - \epsilon)L$:

$$\begin{aligned} \delta_L \phi &= [\Phi]_{L \rightarrow L - \epsilon L} - \Phi = \epsilon y \Phi' - \epsilon L \sum e^{2\pi i n y / L} \partial_L \mathbf{f}_n, \\ \delta_L g_{\mu\nu} &= -2\epsilon y \sigma'_{\mu\nu} - 2\epsilon L \sum e^{2\pi i n y / L} \partial_L \mathbf{s}_n \eta_{\mu\nu}, \\ \delta_L g_{4\mu} &= \delta_L g_{44} = 0. \end{aligned} \quad (19)$$

Though this suggests the need to include periodic perturbations multiplied by linear functions of y , this is not the case since these modes can be absorbed using appropriate coordinate transformations. Under $x^i \rightarrow x^i + \xi^i(x)$,

$$\begin{aligned} \delta_{\text{coord}} g_{ij} &= \xi_{i;j} + \xi_{j;i} + O(\xi^2), \\ \delta_{\text{coord}} \phi &= \xi^i \partial_i \Phi + O(\xi^2), \end{aligned} \quad (20)$$

where a semicolon denotes a covariant derivative, and choosing $\xi_4 = -\epsilon y$, $\xi_\mu = 0$, a combination of (19) and (20) gives

$$\begin{aligned} \delta'_L g_{44} &= -\epsilon; \quad \delta'_L g_{4\mu} = 0; \\ \delta'_L g_{\mu\nu} &= -2\epsilon L \sum e^{2\pi i n y / L} \partial_L \mathbf{s}_n \eta_{\mu\nu}, \\ \delta'_L \phi &= -\epsilon L \sum e^{2\pi i n y / L} \partial_L \mathbf{f}_n, \end{aligned} \quad (21)$$

where $\delta'_L = \delta_L + \delta_{\text{coord}}$. The variations $\delta'_L g_{ij}$, $\delta'_L \phi$ are then equivalent to (19) and are periodic in y .

Hence I will look for solutions to (5) of the form

$$\begin{aligned} \phi &= \Phi(y) + e^{ip \cdot x} |m|^{-1/2} \chi(y), \quad p \cdot x = p_\mu x^\mu; \\ h_{ij} &= H_{ij}(y) + e^{ip \cdot x} \gamma_{ij}(y), \end{aligned} \quad (22)$$

keeping only first-order terms in the perturbations χ , γ_{ij} (the matrix $|m|^{-1/2}$ – cf. eq. 15 – is included for later convenience).

In order to simplify the calculations it is convenient to choose coordinates such that

$$\gamma_{4\mu} = 0; \quad \gamma_i^i = 0. \quad (23)$$

(as before, indices are raised and lowered using the flat-space metric). These conditions, however, do not completely fix the coordinates: using (20) with

$$\xi^4 = iX'(y)e^{ip \cdot x}, \quad \xi^\mu = X(y)p^\mu e^{ip \cdot x}; \quad X'' + p^2 X = 0, \quad X \sim 1/\varphi, \quad (24)$$

yields $\gamma_{\mu\nu} \rightarrow \gamma_{\mu\nu} + 2ip_\mu p_\nu X$ and $\gamma_{44} \rightarrow \gamma_{44} + 2iX''$, which preserve (23).

Expanding

$$\gamma_{\mu\nu} = \frac{p_\mu p_\nu}{p^2} \Gamma_L + \left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \Gamma_T + \tilde{\gamma}_{\mu\nu}; \quad p^\mu p^\nu \tilde{\gamma}_{\mu\nu} = \eta^{\mu\nu} \tilde{\gamma}_{\mu\nu} = 0, \quad (25)$$

where $p^2 = \eta^{\mu\nu} p_\mu p_\nu$, and substituting (22) in (5), yields

$$\begin{aligned} \Gamma_T &= -\vartheta, & \Gamma_L + \gamma_{44} &= 3\vartheta, \\ \tilde{\gamma}_{\mu\nu}'' - p^2 \tilde{\gamma}_{\mu\nu} &= 0, & \gamma_{44}'' + p^2 \gamma_{44} &= 4\vartheta'' - p^2 \vartheta, \end{aligned} \quad (26)$$

and

$$H_0 \chi = -p^2 \chi; \quad H_0 = -\frac{d^2}{dy^2} + SU, \quad (27)$$

where the first equation in (26) enforces the second condition in (23), S is defined in (15), and

$$\vartheta = \frac{4\bar{\xi}}{3|\mathbf{a}|}\chi_1, \quad U_{rs}(y) = -\frac{1}{\sqrt{|m_r m_s|}} \left(\frac{\partial^2 V}{\partial \phi_r \partial \phi_s} \right)_{\phi=\Phi}. \quad (28)$$

Destabilizing modes correspond to periodic (in y) solutions to these equations when $p^2 > 0$; in this case (26) require $\tilde{\gamma}_{\mu\nu} = 0$, as otherwise the solutions would not be periodic. Note also that the modes $\chi = \tilde{\gamma}_{\mu\nu} = \Gamma_T = 0$, $\Gamma_L = -\gamma_{44}$, with γ_{44} obeying the homogeneous equation $\gamma''_{44} + p^2 \gamma_{44} = 0$, are apparently destabilizing, but these modes are coordinate artifacts that can be eliminated by a transformation of the form (24).

Since the homogeneous solution to the γ_{44} equation in (26) can be eliminated using (24), we can assume that γ_{44} is determined by $\vartheta \propto \mathbf{a} \cdot \chi$. The stability of the background configuration then depends only on whether (27), the equation satisfied by χ , has periodic solutions only for $p^2 < 0$. The analysis of this equation is complicated by the fact that SU is not Hermitian (for the usual definition of the inner product), so that its similarity to the Schrödinger equation is not very useful in this case². Instead I argue as follows

The general solution to the equation $m\Phi'' = \mathbf{F}$ in (7) depends on $2N$ constants (*e.g.* the values of Φ and Φ' at $y = 0$). The periodicity requirement (10) introduces N restrictions and one new parameter, since L is not fixed *a priori*. Finally, (13) imposes one additional constraint. As a result the general periodic solution will depend on N arbitrary parameters (including L); since the equation of motion does not depend explicitly on y , one of these parameters can be taken as some initial value y_0 . The general periodic solution can then be written

$$\Phi = \Phi(y - y_0; \mathbf{c}; L), \quad \mathbf{c} = (c_1, \dots, c_{N-2}). \quad (29)$$

Let now

$$\chi^{(0)} = \partial_{y_0} \Phi, \quad \chi^{(N-1)} = \frac{1}{\epsilon} \delta'_L \Phi, \quad \chi^{(i)} = \partial_{c_i} \Phi, \quad i = 1, \dots, N-2; \quad (30)$$

where δ'_L is defined below (21). Then, by taking the appropriate derivatives of the equation of motion (7), it is easy to see that

$$H_0 \chi^{(i)} = 0, \quad i = 0, \dots, N-1. \quad (31)$$

Since the constants y_0 , L and \mathbf{c} are independent, the modes $\chi^{(i)}$, $i = 0, \dots, N-1$ will be linearly independent (and periodic).

The equation $H_0 \chi = 0$ is a linear, second-order differential equation for the component functions χ_r , so the general solution will contain $2N$ arbitrary constants. The periodicity condition, however, imposes N constraints (note that now L is fixed by the background solution), so that the general periodic solution will contain only N independent constants. Then, since the equation is linear, any periodic solution to (27) with $p^2 = 0$ can be written as a linear combination of the $\chi^{(i)}$.

Consider next those background solutions that remain near an extremum Φ_{extr} of the potential V (the presence of an extremum is a necessary condition for the existence of periodic solutions (10); when $m_1 < 0$, as for the cases of interest, this extremum is a saddle point). For these cases the relevant equations are, approximately,

$$\tilde{\Phi}'' = S\bar{U}\tilde{\Phi}, \quad -\chi'' + S\bar{U}\chi = -p^2\chi; \quad (32)$$

where

$$\tilde{\Phi}_r = |m_r|^{1/2} (\Phi - \Phi_{\text{extr}})_r; \quad \bar{U}_{rs} = -|m_r m_s|^{-1/2} \left(\frac{\partial^2 V}{\partial \phi_r \partial \phi_s} \right)_{\phi=\Phi_{\text{extr}}}. \quad (33)$$

Now, for there to be periodic solutions $\tilde{\Phi}$, $S\bar{U}$ must have at least one negative eigenvalue $-\omega^2$ (ω real); denoting by \mathbf{v} the corresponding eigenvector, it follows that the y -independent choice $\chi = \mathbf{v}$ is a solution to the χ equation with $p^2 = \omega^2 > 0$, which is then a destabilizing mode. It follows that all small amplitude solutions are unstable.

Now consider a small amplitude solution $\Phi(y - y'_0; \mathbf{c}'; L')$, another arbitrary solution $\Phi(y - y''_0; \mathbf{c}''; L'')$, and a smooth path in parameter space $\{y_0(s), \mathbf{c}(s), L(s)\}$, $0 \leq s \leq 1$, such that $\{y_0(0), \mathbf{c}(0), L(0)\} = \{y'_0, \mathbf{c}', L'\}$ and

² H_0 is Hermitian under the inner product $\langle \chi_1 | \chi_2 \rangle_S = \oint dy \chi_1^\dagger S \chi_2$, but, though this can be used to show that p^2 must be real, it does not provide information about its sign since this inner product is not positive definite, and the arguments based on the Sturm-Liouville theory [18, 23] are not applicable.

$\{y_0(1), \mathbf{c}(1), L(1)\} = \{y_0'', \mathbf{c}'', L''\}$. Each value of s defines a periodic solution Φ with period $L(s)$, from which a corresponding U is constructed. As a result all the modes χ will depend smoothly on s , and so will the eigenvalues $-p^2$; in particular, the $p^2 = 0$ subspace is N dimensional for all s . For $s = 0$ we know there is at least one mode with $p^2 > 0$, but then this mode must remain destabilizing for all s , for if it were to change from destabilizing to stable, there would be a value \bar{s} at which its eigenvalue p^2 would vanish, so that for $s = \bar{s}$ the $p^2 = 0$ subspace would have dimension $N + 1$, which we saw above is impossible. It follows that the corresponding solution $\Phi(y - y_0''; \mathbf{c}''; L'')$ is also unstable, and since the parameters $\{y_0'', \mathbf{c}'', L''\}$ are arbitrary, it follows that all background solutions in the pure scalar-gravity model are unstable.

There is a subtlety in the above argument. One can easily imagine potentials with two or more extrema, each with different number of destabilizing modes, yet the above argument seems to indicate that the number of destabilizing modes cannot change. This apparent contradiction is resolved by noting that there is no periodic solution that can interpolate between the corresponding small-amplitude solutions. Imagine a potential with 2 extrema and small-amplitude solution near each, denoted by $\Phi^{(1)}(y)$ and $\Phi^{(2)}(y)$, and for which the corresponding χ equations in (32) have different numbers of destabilizing modes. Then for any periodic function $\Phi(y, s)$, $0 \leq s \leq 1$, of period $L(s)$, that solves (7) for each s , and such that $\Phi^{(1)} = \Phi|_{s=0}$, $\Phi^{(2)} = \Phi|_{s=1}$, there will be an intermediate value $0 < s_\infty < 1$ such that $L(s) \rightarrow \infty$ as $s \rightarrow s_\infty$. Thus, to every extremum one can associated a “region of influence” determined by all solutions that can be reached by an interpolation with $L(s)$ finite for all s ; the above argument shows that all such solutions are unstable. I will assume that all periodic solutions can be characterized in this way, that is, that any periodic solution can be “deformed” into a small amplitude solution near an extremum of V while keeping the period finite. These results extend the arguments of [18, 19] to the class of models considered here.

Similar considerations can be followed in case the potential is not quadratic near the extremum.

IV. ANTISYMMETRIC TENSOR STABILIZATION MECHANISM

The instability of the solutions to (7) is reminiscent of the well-known instability of soliton-like solutions in more than $1 + 1$ dimensions [22]. In the soliton case stable solutions are obtained by introducing gauge fields, here I will pursue a different approach based on the introduction of an antisymmetric tensor field A that can propagate in the bulk, and which has the following interactions:

$$\begin{aligned} \mathcal{L}_A &= -\frac{1}{12}g^{ij}g^{kl}g^{mn}A_{ikm}A_{jln} - \frac{1}{4}\kappa^2g^{ij}g^{kl}A_{ik}A_{jl} + \frac{1}{2}\kappa g^{ij}g^{kl}J_{ik}A_{jl}; \\ J_{ik} &= \sum_{r,s} |m_r m_s|^{1/2} \lambda_{rs} \phi_{r,i} \phi_{s,k}, \end{aligned} \quad (34)$$

where $A_{ijk} = 5A_{[ij,k]}$ and $\lambda_{rs} + \lambda_{sr} = 0$ (the couplings λ could depend on ϕ also, but such a case will not be considered here). When considering the quantum aspects of this model it is convenient to rewrite \mathcal{L}_A using an auxiliary Stückelberg-like vector field (see, for example, [26]); the expression (34) then corresponds to the “unitary” gauge where this auxiliary field vanishes. The presence of higher-derivative terms, while innocuous classically, can have dire quantum effects; still, since these theories have an intrinsic UV cutoff, these are avoided with appropriate constraints on the scales related to the couplings λ (see sect. V). Antisymmetric tensors have been considered previously in the literature in RS-like models with or without fundamental scalars [27], in non-periodic thick-brane models with weak scalar-tensor couplings [28], and in exotic-Lagrangian models addressing the self-tuning of the cosmological constant in RS-like models [29]. The type of models described by (34) has apparently not been discussed previously within the present context.

I will assume that $\kappa = O(\varphi^{2/3})$ so that $A \sim 1/\varphi$; then

$$\mathcal{L}_A = -\frac{1}{4}(\kappa A - J)_{ij}(\kappa A - J)^{ij} + \frac{1}{4}J_{ij}J^{ij} + O(1/\varphi), \quad (35)$$

where indices are raised and lowered using the flat space metric η_{ij} . In this case the field A can be integrated out and one can work instead with the effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \frac{1}{4}J_{ij}J^{ij}, \quad (36)$$

up to corrections of order $1/\varphi$.

The addition of \mathcal{L}_{eff} does not change the background equations (to lowest order) so that the results of sections II A-II C are not modified; in terms of the antisymmetric tensor field this implies that it vanishes in the background configuration.

A. Stability with antisymmetric tensors

The perturbation equation is modified by the addition of (36). Instead of (27) one gets

$$H_0\chi = -p^2[\chi - S\mathbf{B}(\mathbf{B} \cdot \chi)] , \quad B_r = \sum_s |m_s|^{1/2} \lambda_{rs} \Phi'_s . \quad (37)$$

As before the perturbative stability of the background configuration $\tilde{\Phi}$ is guaranteed if this equation has no solutions for $p^2 > 0$.

Given the form of (37), it is clear that the modes $\chi^{(i)}$ defined in (30) again provide N solutions corresponding to $p^2 = 0$. The argument of section III then implies that the background configurations will be stable provided the small-amplitude background configurations near an extremum are stable; this can be realized when $\mathbf{B} \neq 0$.

For background configurations that remain near an extremum of the potential V we still have $\tilde{\Phi}'' = S\bar{U}\tilde{\Phi}$ as in (32), but the equation for χ becomes

$$-\chi'' + S\bar{U}\chi = -p^2\chi + p^2 S\bar{\mathbf{B}}(\bar{\mathbf{B}} \cdot \chi) , \quad \bar{B}_r = \sum_s \lambda_{rs} \tilde{\Phi}'_s . \quad (38)$$

In order to have periodic solutions $\tilde{\Phi}$, $S\bar{U}$ must have one or more negative eigenvalues $-\omega_a^2$, with \mathbf{v}_a the corresponding eigenvectors; however, the constant modes $\chi = \mathbf{v}_a$ are not solutions of (38) when $\mathbf{B} \neq 0$.

I will now consider the case where the potential takes the form (11), with u_r of the shape given in Fig. 1 for all r . Then each $\tilde{\Phi}'_r$ will be approximately a square wave, so that I can approximate $(\tilde{\Phi}'_r)^2 \rightarrow a_r = \text{constant}$, and the sum rule (13) becomes

$$a_1 = \sum_{r>1} a_r . \quad (39)$$

In addition, $(S\bar{U})_{rs} = \delta_{rs}\mathcal{V}_r$ is diagonal, and each \mathcal{V}_r can be approximated by a delta-function comb, with negative magnitude and half the period of $\tilde{\Phi}_r$.

I will now make the simplifying assumption that the periods of $\tilde{\Phi}_r$ for $r > 1$ are much smaller than that of $\tilde{\Phi}_1$; for example, if ℓ denotes the period of $\tilde{\Phi}_1$, that of $\tilde{\Phi}_r$, $r > 1$ can be chosen to be ℓ/n_r with n_r a large prime number; the period L in (10) then equals $(\prod n_r)\ell$. In this case $S\bar{U}$ and $\bar{\mathbf{B}} \otimes \bar{\mathbf{B}}$ in (38) will have entries that are slowly varying and others that vary very rapidly, and this equation can be treated using the procedure described in [30]: up to corrections of order $1/n_r$, the solution can be obtained by considering only the slowly varying terms in (38):

$$-\chi''_r - \delta_{r,1}\mathcal{V}_1\chi_1 = -p^2\chi_r + p^2 S_r \sum_s \mathcal{B}_{rs}\chi_s \quad (40)$$

(no sum over r) where, as before, $S_1 = -1$, $S_{r>1} = +1$, and

$$\mathcal{B}_{rs} = \sum_{u>1} (\lambda_{r1}\lambda_{s1} + \lambda_{ru}\lambda_{su}) a_u . \quad (41)$$

The term containing \mathcal{B} in (40) is generated by averaging the rapidly varying terms in (38) over periods $\ll \ell$.

The equation for χ_r can now be analyzed using standard techniques (see e.g. [31]): denoting $\mathbf{X} = (\chi, \chi')^t$ there will be a non-singular matrix T such that $\mathbf{X}(y+L) = T\mathbf{X}(y)$, and the system will support destabilizing modes if, for some *positive* p^2 , T has an eigenvalue equal to one (which corresponds to χ being periodic). In determining the conditions under which this occurs one is interested only in modes for which $|p^2| < \Lambda^2$, or, equivalently, $[p\ell/(4\pi)]^2 \leq [\Lambda L/(4\pi\mathcal{N})]^2 = \wp$, where $\mathcal{N} = \prod n_r$ and ℓ/n_r is the period of $\tilde{\Phi}_r$, defined above; with appropriate choice of these parameters one can insure $\wp = \mathcal{O}(10)$. In this case there are ranges of parameters where not destabilizing modes are present; Fig. 2 contains an example for fixed λ_{rs} , a_r and \mathcal{V}_1 where the matrix $T \neq 1$ for $p^2 > 0$, while Fig. 3 gives, for fixed $\lambda_{r,s}$, the values of $a_{2,3}$ where no destabilizing modes occur (within the allowed p^2 range). Background configurations for this type of models are then stable in such cases.

Stable solutions occur when the number of scalar fields is ≥ 3 ; I found no stable configurations in more economical models.

V. COMMENTS

The main purpose of the present paper was to provide mechanisms that can overcome the periodicity and stability problems of brane-like configurations produced by scalar fields. The model defined by (1) and (34) meets these

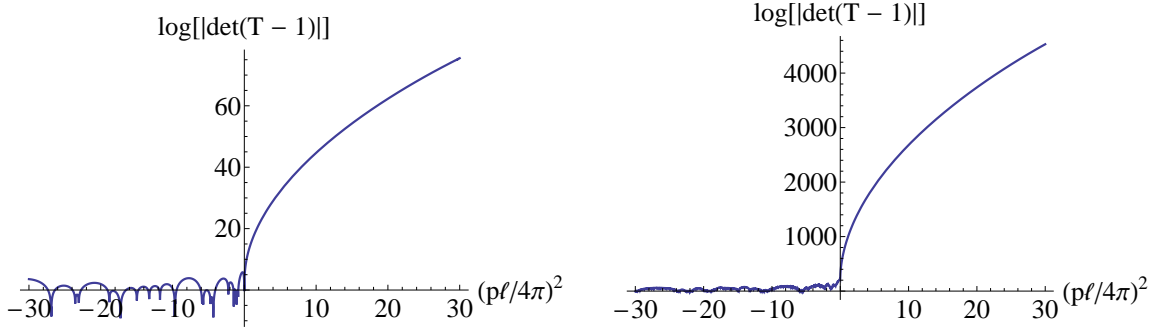


FIG. 2: Value of $\log |\det(T - \mathbb{1})|$ when $\mathcal{N} = 1$ (left) and $\mathcal{N} = 60$ (right). In both cases $\lambda_{12} = -0.2$, $\lambda_{13} = -0.4$, $\lambda_{23} = 0.3$, $a_2 = 0.05$, $a_3 = 0.7$, and the amplitude of \mathcal{V}_1 equals $28\pi/\ell$. Periodic solutions occur for $p^2 \leq 0$ and do not correspond to destabilizing modes.

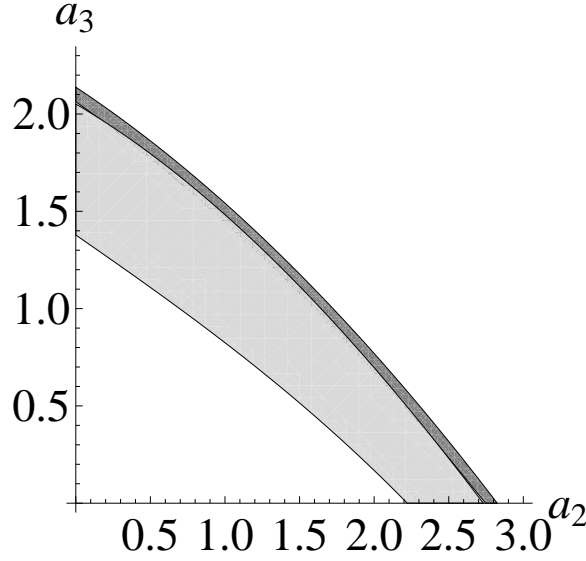


FIG. 3: Values of $a_{2,3}$ for which (40) has no destabilizing modes. Light gray: $\varphi < 30$ and the amplitude of \mathcal{V}_1 equals $28\pi/\ell$; dark gray: $\varphi < 20$ and the amplitude of \mathcal{V}_1 equals $4\pi/\ell$. In both cases $\lambda_{12} = -0.2$, $\lambda_{13} = -0.4$, $\lambda_{23} = 0.3$.

requirements by introducing a coupling of the scalars to the Ricci scalar and to an antisymmetric tensor field that can propagate in the bulk. For appropriate choices of scalar potential the background configurations mimic that of the RS model but require the presence of ≥ 3 scalar fields with modes of very different though commensurate periods (an investigation of whether this hierarchy can be maintained naturally lies beyond the scope of this paper).

As mentioned in section I these models contain a UV cutoff scale Λ ; when \mathbb{B} is compact they also contain another high-energy scale, the compactification radius L . An immediate concern is whether one can naturally assume $\Lambda \gg 1/L$, for otherwise the model cannot accurately describe the dynamics associated with the compact directions. This question can be investigated using an extension of naive dimensional analysis [4]. For example, a straightforward estimate for 5-dimensional gauge theories gives $\Lambda \sim 24\pi^3/g_5^2$, where g_5 is the 5-dimensional gauge coupling constant (which is dimensional). The 4-dimensional gauge coupling is $g \sim g_5/\sqrt{L}$, whence $L\Lambda \sim 24\pi^3/g^2$, which indicates that such models are reliable at energies above the compactification scale provided $6\pi^2 \gg g^2/(4\pi)$. For compact \mathbb{B} the scale Λ also provides a cutoff for the order of the KK excitations that need to be included in the theory; the effects of higher order modes are absorbed into a renormalization of the various operator coefficients.

When the antisymmetric tensor field is introduced, additional constraints appear, such as, for example, those derived from the unitarity constraint in $\phi\phi \rightarrow \phi\phi$ scattering, which demands (at tree-level) that the scale associated with λ be below $\sim \Lambda/(2\pi)$; this restriction also insures that the effective Lagrangian (36) does not generate undesirable poles

in the scalar propagators.

In the above models there are no exponentially destabilizing modes below scale Λ . There *are*, however, zero modes obtained by taking derivatives with respect to the parameters of the background configuration (see Eq. 30). Perturbations along these zero modes then correspond to small deformations of the background configuration parameters, under which its period and orbits suffer small changes, but and do not result in an instability.

Phenomenologically viable models constructed along these lines must include, in addition, the possibility of orbifolding. In this case it is most convenient to assume that the potential allows solutions with definite parity under $y \leftrightarrow -y$. This can be implemented without additional complications.

It is worth noting that the present models cannot be stabilized by the 1-loop effective potential [25]. This is because in the present case the tree-level potential U supports at least one destabilizing mode (whose amplitude increases exponentially with time), not merely a neutral one. Loop corrections are of course present, but they are subdominant.

Appendix A: Alternative interpretation of the scalar equations of motion

When $\bar{\xi} > 3/16$, the presence of a negative sign in S (cf. eq. 15) suggests an alternative description of the equations for Φ as a geodesic equation, and that for χ as a geodesic deviation equation.

To see this consider an $N + 1$ -dimensional space with coordinates $\Phi_1, \dots, \Phi_N, \theta$ and metric

$$\Gamma_{RS} = \text{diag} (m_1, m_2, \dots, m_N, 1/V(\Phi)) . \quad (\text{A1})$$

I will denote by r, s , etc. the indices corresponding to the first N coordinates, then $\Gamma_{rs} = m_r \delta_{rs}$. The geodesic equations associated with this metric are

$$\frac{d^2 \Phi_r}{dp^2} = -\frac{1}{2m_r V^2} \left(\frac{\partial V}{\partial \Phi_r} \right) \left(\frac{d\theta}{dp} \right)^2 ; \quad \frac{d^2 \theta}{dp^2} = \frac{1}{V} \sum_r \left(\frac{\partial V}{\partial \Phi_r} \right) \left(\frac{d\Phi_r}{dp} \right) \left(\frac{d\theta}{dp} \right) ; \quad (\text{A2})$$

where p is an affine parameter.

The second equation can be immediately integrated: $d\theta/dp = cV$ where c is a constant; substituting into the equation for the Φ_r gives

$$\frac{2}{c^2} \frac{d^2 \Phi_r}{dp^2} = - \left(\frac{\partial V}{\partial \Phi_r} \right) , \quad (\text{A3})$$

that reduces to the second-order equation for Φ in (7) provided y is identified with $cp/\sqrt{2}$.

The zero-energy condition in (7) corresponds to the requirement that this be a null geodesic:

$$\sum_{rs} \frac{d\Phi_r}{dp} \frac{d\Phi_s}{dp} \Gamma_{rs} + \frac{1}{V} \left(\frac{d\theta}{dp} \right)^2 = c^2 \left\{ \frac{1}{2} \Phi' m \Phi' + V(\Phi) \right\} = 0 , \quad (\text{A4})$$

where a prime denotes a y derivative. It is also clear that the equation for χ is equivalent to the geodesic deviation equation associated with the metric (A1).

Appendix B: Background solutions for models with a single scalar field.

When there is a single scalar field the background solution takes the form

$$\begin{aligned} \tilde{\phi} &= \Phi + \Theta/\varphi + \dots \\ g_{\mu\nu} &= (1 - 2\sigma/\varphi - 2\tau/\varphi^2 + \dots) \eta_{\mu\nu} \end{aligned} \quad (\text{B1})$$

with $g_{44} = 1$, $g_{4\mu} = 0$.

For this simple case the field equation is redundant; while the Einstein equations give

$$\begin{aligned} O(\varphi) : \quad & 2\zeta\Phi'' - 3\sigma'' = 0 , \\ O(1) : \quad & V(\Phi) - \frac{1}{2}m\Phi'^2 - \frac{2\zeta}{3\zeta}(2\zeta\Phi' - 3\sigma')^2 = 0 , \\ & 2\Theta'' - \frac{3}{\zeta}\tau'' - 6\Phi\sigma'' + 2\Phi'\sigma' - \frac{1}{\zeta}\Phi'^2 + (\Phi^2)'' = 0 , \end{aligned}$$

$$O(1/\varphi): \quad 4\xi\Phi(2\Phi-3\sigma)'\sigma' - \frac{12\xi}{\zeta}\sigma'\tau' + (8\xi\tau - \Theta)'\Phi' + 8\xi\sigma'\Theta' + \Theta\left(\frac{\partial V}{\partial\phi}\right)_\Phi = 0 \quad (\text{B2})$$

(I omitted an additional equation to order $1/\varphi$ that determines the correction of order $1/\varphi^3$ to the metric), where

$$\bar{\xi} = \frac{\xi^2}{\xi - 4M_{\text{Pl}}^3/\varphi^2}, \quad \zeta = \bar{\xi}/\xi, \quad m = 1 - \frac{16}{3}\bar{\xi}. \quad (\text{B3})$$

The $O(\varphi)$ equation together with the periodicity requirement imply $2\zeta\Phi - 3\sigma = \text{constant}$, whence Φ must satisfy

$$\frac{1}{2}m\Phi'^2 + V(\Phi) = 0. \quad (\text{B4})$$

It now proves convenient to write

$$\begin{aligned} \Theta &= \Phi' A(y), \\ \tau &= \frac{1}{2\xi}f(\Phi) + \frac{2\zeta}{3}\left(\frac{1-\zeta}{m} - \frac{1}{2}\right)\Phi^2 + B(y), \end{aligned} \quad (\text{B5})$$

Substituting into (B2) and using (B4) one finds

$$\begin{aligned} \sigma &= \frac{2\zeta}{3}\Phi; \quad A' = \frac{(m-1)(1-\zeta)}{3}\Phi; \\ B'' &= \frac{2\zeta}{3}A\Phi'''; \quad f'' + \frac{1}{2}\frac{V'}{V}f' + \frac{2\zeta}{3}\left[m + \frac{32\xi^2}{3}\frac{\zeta(1-\zeta)}{m}\right] = 0; \end{aligned} \quad (\text{B6})$$

where a prime denotes a derivative with respect to the argument. These equations can be solved by quadratures; in particular,

$$f(\phi) = -\frac{2\zeta}{9m}\left[3m^2 + 32\bar{\xi}(\xi - \bar{\xi})\right]\int_0^\phi d\lambda \int_0^\lambda d\gamma \sqrt{\frac{V(\gamma)}{V(\lambda)}}. \quad (\text{B7})$$

Higher orders can be dealt with similarly. For more than one field the higher-order corrections to (7) cannot, in general, be cast into such comparatively simple expressions.

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